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Procedure for Initializing Interface Stiffnesses Based on Sensitizing Inputs

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Parameter identification and optimization procedures often fail when the target model responses are insensitive to variations in the selected design variables (e.g., Young's modulus, plate thickness, etc.) of the nominal model. This insensitivity simply indicates that the kinetic and strain energies associated with a group of elements relative to a given variable are negligible with respect to the total energies of the complete structure. Two independent conditions can lead to this situation. First, the model displacement fields are localized and do not attain the design variable in question. Second, the nominal properties of both the elements governed by the design variable and environing structure are such that no load paths cross through the zone in question. We are interested in determining the domain of sensitivity of a design variable so that it can be reinitialized in preparation for an optimization procedure. The proposed methodology is based on the application of sensitizing inputs and uses the ratio of local to global energies to redefined nominal stiffness properties. The procedure is validated with academic and industrial structures.

Nomenclature

diag(A)	=	vector of the diagonal terms of the matrix A
$E_{i\nu}$	=	strain energy of the group of elements associated with the parameter p_i relative to the response $\boldsymbol{y}_\nu$
E_ν	=	total strain energy in the model relative to the response $\boldsymbol{y}_\nu$
F	=	matrix of forces
$\boldsymbol{f}_{i\nu}$	=	νth vector of force relative to the parameter p_i
$\Vert\boldsymbol{f}\Vert_2$	=	norm 2 of the vector $\boldsymbol{f}$
K, M	=	stiffness and mass matrices, respectively
K_i	=	stiffness matrix containing the elements corresponding to the parameter p_i
k_i^l	=	stiffness of the ith connecting element at the iteration l
N	=	number of connecting stiffnesses to reinitialize
p_i	=	parameter i
$\boldsymbol{y}_\nu$	=	νth displacement field
Z	=	dynamic stiffness matrix
α	=	weight coefficient of parameters

I. Introduction

PARAMETER identification and optimization procedures often fail when the target model responses are insensitive to variations in the selected design variables (e.g., Young's modulus, plate thickness, etc.) of the nominal model. This insensitivity simply indicates that the kinetic and strain energies associated with a group of elements relative to a given variable are negligible with respect to the total energies of the complete structure. Two independent conditions can lead to this situation. First, the model displacement fields are localized and do not attain the design variable in question. Second, the nominal properties of both the elements governed by the design variable and environing structure are such that no load paths cross through the zone in question. In this paper, we are interested in determining the domain of sensitivity of a design variable so that it can be reinitialized in preparation for an optimization procedure.

Finite element models of complex mechanical structures are often based on a simplification of the real geometry. This is especially true for interface components (bolts, spot welds, crimps, etc.) whose detailed representation would otherwise lead to excessively large model orders. These components are usually represented by local Hooke stiffnesses interposed between the nodal degrees of freedom in question. Although this simplification proves to be quite practical, it leads to an additional difficulty, namely, the problem of estimating the nominal values of these local stiffnesses in preparation for a parametric optimization or identification. Moreover, this modeling task proves to be excessively important because of the potential influence of the interface characteristics on the global behavior of the structure. A modeler will typically have access to a database of nominal values obtained either experimentally or through detailed analysis of their assemblies, but in many cases the initialization and adjustment of these stiffnesses is performed manually.

We present in this paper an automatic procedure for reinitializing the values of the Hooke coefficients for a set of local spring elements. The proposed methodology is based on the application of sensitizing static inputs that allow the relative energy in a local spring to be compared to the total energy in the structure. This allows the stiffness of each spring to be adjusted independently as a function of the stiffness of the surrounding structural zone. Moreover, the use of sensitizing inputs allows the influence of a given spring on the global static response to be rapidly estimated with high accuracy.

II. Mathematical Formulation

A. Hypotheses

The formulation of the proposed methodology will be limited here to the linear conservative elastodynamic behavior of mechanical structures. The discretized dynamic equilibrium equation in the frequency domain can be expressed as

$$Z(\omega)\boldsymbol{y}(\omega) = \boldsymbol{f}(\omega) \tag{1}$$

where $\boldsymbol{y}, \boldsymbol{f} \in \Re^{N,1}$ are the output and input vectors, respectively; $Z(\omega) \triangleq (K - \omega^2 M)$ is the dynamic stiffness matrix as a function ω; the angular frequency, $M, K \in \Re^{N,N}$, both constant, symmetric and positive definite.

Inversely, the output behavior of the nominal model with respect to a harmonic input excitation can be expressed by

$$\boldsymbol{y}(\omega) = \Gamma(\omega)\boldsymbol{f} \tag{2}$$

where $\Gamma(\omega) = Z(\omega)^{-1} \in \Re^{N,N}$ is the dynamic flexibility matrix.

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B. General Problem Statement

Definition 1: Let $K(\alpha p_i) \in \Re^{N,N}$ be the global stiffness matrix resulting from the modification of a design parameter p_i (e.g., plate thickness or offset, Hooke's coefficient, beam properties) by a coefficient α, $0 \leq \alpha \leq \infty$, and $K_i(\alpha p_i) \in \Re^{N,N}$ be the stiffness matrix containing the elements corresponding to the parameter p_i (generally sparse) resulting from the modification of a design parameter p_i.

Definition 2: The sensitivity $S_{i\nu}$ of a parameter p_i with respect to a response $\boldsymbol{y}_\nu$ is defined by

$$S_{i\nu} = E_{i\nu}/E_\nu \tag{3}$$

where $E_{i\nu} = \boldsymbol{y}_\nu^T K_i(\alpha p_i)\boldsymbol{y}_\nu$ and $E_\nu = \boldsymbol{y}_\nu^T K(\alpha p_i)\boldsymbol{y}_\nu$ correspond to the strain energy of the group of elements associated with the parameter p_i and the total strain energy in the model, respectively.

Definition 3: An input vector $\boldsymbol{f}$ is said to sensitize the parameter p_i if and only if $S_{i\nu} > \varepsilon$, where ε is an arbitrary scalar such that $0 \leq \varepsilon \leq 1$.

The problem we seek to solve can be formulated as follows. Determine the range of values of the parameter p_i such that it is sensitized by the system input $\boldsymbol{f}$ at a frequency ω.

Remark: Standard first-order output sensitivities based on modal properties (eigenvectors or eigenmodes) are generally not suitable for the application at hand while a given modal property might not be sensitive to a parameter p_i; this can be an artifact of the corresponding displacement field and not the nominal value of p_i. To avoid this source of uncertainty and to guarantee that we are in the appropriate conditions to judge the influence of p_i, we have chosen to apply the notion of sensitizing inputs.

C. Figures of Merit

The following figures of merit can be evaluated to characterize the relative sensitivity of a design parameter p_i:

$$\mathcal{M}_i^1 = \sum_{\nu=1}^{m} \frac{E_{i\nu}}{E_\nu} \tag{4}$$

$$\mathcal{M}_i^2 = \sum_{\nu=1}^{m} \frac{E_{i\nu}}{\max\limits_{\alpha \in \mathcal{A}} (E_\nu)} \tag{5}$$

where m is the number of displacement fields, $\mathcal{A}$ is the set of all α values tested, $\mathcal{M}_i^1$ is a measure of the relative strain energy in the design parameter with respect to the total strain energy of the same displacement vector, $\mathcal{M}_i^2$ is a measure of the total strain energy in the structure with respect to the maximum total strain energy obtained over α, and

$$\begin{aligned} E_{i\nu} &= \max\left\{\mathrm{diag}\left[X_\nu^T(\alpha p_i)K_i(\alpha p_i)X_\nu(\alpha p_i)\right]\right\} \\ &= \max\left\{\mathrm{diag}\left[F^T K^{-1}(\alpha p_i)K_i(\alpha p_i)K^{-1}(\alpha p_i)F\right]\right\} \end{aligned} \tag{6}$$

$$E_\nu = \max\left\{\mathrm{diag}\left[F^T K^{-1}(\alpha p_i)F\right]\right\} \tag{7}$$

where X_ν is the displacement matrix.

D. Sensitizing Inputs

The notion of sensitizing inputs has been used in the past for the identification of mechanical systems by several authors.[1,2] In the present case, the displacement fields of the model that are typically available, such as eigenmodes or frequency responses, are not necessarily sensitive to the stiffness parameters in question. This might be so for two reasons. First, the available displacement fields might simply not lead to local deformation in the region englobing the stiffness parameter of interest, thus rendering them ineffective for solving the problem at hand. Second, the nominal value of the stiffness parameter in question might simply be too large or too small to influence the global structural response. The objective of generating sensitizing inputs is to avoid the first difficulty by ensuring that the displacement field is appropriate for testing the sensitivity of the stiffness parameter in question.

Let $H_i \in \Re^{N,N_i}$ be the row restriction of the unit matrix I_N to the N_i model degrees of freedom for which the matrix K_i has nonzero elements. Construct a linearly independent set of sensitizing input vectors $F_i = [\cdots \boldsymbol{f}_{i\nu} \cdots]$, satisfying

$$\min_{f_{i\nu}} \boldsymbol{y}_{i\nu}^T K \boldsymbol{y}_{i\nu} \tag{8}$$

where

$$\boldsymbol{y}_{i\nu} = \Gamma(\omega)H_i\boldsymbol{f}_{i\nu} \tag{9}$$

and $\boldsymbol{f}_{i\nu} \in \Re^{N_i,1}$ subject to the constraint

$$\|\boldsymbol{f}_{i\nu}\|_2 = 1 \tag{10}$$

The basis F_i can be shown to be the solution to the eigenvalue problem:

$$\left[H_i^T\Gamma(\omega)K\Gamma(\omega)H_i - \sigma_{i\nu}I\right]\boldsymbol{f}_{i\nu} = 0 \tag{11}$$

Remarks:

1) The energy function defined in the optimization problem (8) ensures a relatively simple low-energy deformation shape.

2) The constraint equation (10) provides a convenient way to bound the force levels and naturally leads to the solution of the eigenvector problem (11).

3) The eigenvalue $\sigma_{i\nu}$ represents the strain energy in the structure as a result of the input vector $\boldsymbol{f}_{i\nu}$.

4) Restricting the nonzero input components to the N_i modified degree of freedom is motivated on physical grounds and ensures that the parameter is effectively sensitized by the input vector.

E. Model-Order Reduction

Although the figures of merit could theoretically be calculated on the basis of exact calculations with the global model, it is more realistic to perform a model reduction in order to obtain reasonable calculation times. Indeed, an exact evaluation of $\boldsymbol{y}_{i\nu}(\alpha, \omega)$ requires the solution of

$$[Z(\omega) + \Delta K_i(\alpha)]\boldsymbol{y}_{i\nu} = H_i\boldsymbol{f}_{i\nu} \tag{12}$$

It proves to be efficient[3] to solve the reduced-order system obtained by writing $\boldsymbol{y}_{i\nu} = Bq$, where $B = \Gamma(\omega)H_iF_i \in \Re^{N,N_i}$; hence,

$$B^T[Z(\omega) + \Delta K_i(\alpha)]Bq_{i\nu} = B^TH_i\boldsymbol{f}_{i\nu} \tag{13}$$

Remarks:

1) This reduced-order system is of dimension $N_i \ll N$.

2) In practice, one needs only to evaluate the evolution of the figures of merit for one or two sensitizing inputs.

3) The figures of merit are evaluated for a range α, typically $10^{-5} \leq \alpha \leq 10^5$.

F. Sensitizing Interface Elements

The general methodology just described can readily be applied to a domain composed of simple stiffness elements. In this case, an interface spring possesses two nodes with one degree of freedom (DOF) node; hence, the reduced model is of order 2. The sensitizing inputs can be expressed as follows. The static equilibrium problem can be written as

$$K\boldsymbol{y} = \boldsymbol{f} \tag{14}$$

Let k_{ij} be an interface stiffness element between the ith and jth DOF of the model. Define two linearly independent input vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ such that

$$\boldsymbol{f}_1 = e_i + e_j \tag{15}$$

$$\boldsymbol{f}_2 = e_i - e_j \tag{16}$$

where e_i, e_j correspond to the ith and jth columns of the unit matrix of order N. Then,

$$x_1 = K^{-1} f_1 \tag{17}$$

$$x_2 = K^{-1} f_2 \tag{18}$$

Let $\tilde{K} \in \Re^{N,N}$ be the modified stiffness matrix given by

$$\tilde{K} = K + \alpha k_{ij} \, L_{ij} \tag{19}$$

where α is a scalar such that $\alpha \geq -1$, k_{ij} is a perturbation to the initial local stiffness, and $L_{ij} \in \Re^{N,N}$ is a diadic matrix defined by

$$L_{ij} = (e_i - e_j)(e_i - e_j)^T \tag{20}$$

The basis $B = [x_1 \;\; x_2] \in \Re^{N,2}$ is used to reduce the global $N \times N$ system.

III. Numerical Implementation

The proposed methodology allows us to determine the value of stiffnesses that sensitize inputs. The order of magnitude of the stiffnesses is in the interval $[10^{-5}, 10^5]$. In the case of nonunique stiffness element, interactions between springs elements can occur. Values that sensitize inputs are under- or overestimated if each spring is treated independently. An iterative algorithm is then used. Static responses, stiffness matrix reduction, figures of merit, and reinitialization are evaluated at each iteration. The convergence is satisfied if two criteria are verified. The classical norm based on the difference of the values for the last two iterations is not usable in this context. Therefore, we use the following convergence criterion in order to ensure low variations of each stiffness between the two last iterations:

$$\max_{i=1,\ldots,N} \left| \log\left(k_i^l\right) - \log\left(k_i^{l-1}\right) \right| \leq \eta \tag{21}$$

where k_i represents the stiffness of the element i, l and $l-1$ are the two last iterations, and η is a tolerance threshold (default value is $\eta = 0.05$). A second convergence criterion is defined for limiting the maximum number of iterations $it_{\max}$:

$$l \leq it_{\max} \tag{22}$$

Remarks: 1) The convergence criterion (21) proves to be easier to implement than a standard $|k_i^l - k_i^{l-1}| \leq \eta$. This is because the values of k_i^l, for $l = 1$ to $it_{\max}$, can vary by many orders of magnitude, and hence η tends to be difficult to define a priori. The criterion (21) allows convergence to be defined as a variation on a power of 10.

2) Convergence problems can occur in the following two cases: a) when a spring element is grounded on one end and connected to an otherwise free structure on the other or b) when a spring element or a group of spring elements connects two free structures. In both cases, the sensitizing forces concentrate all (or nearly all) of the strain energy in the stiffness elements to be reinitialized and little to none in the rest of the structure. Both cases can be avoided simply be defining appropriate boundary conditions.

3) The reinitialization procedure has been implemented in the MATLAB®-based dynamic analysis environment ÆSOP (Analytico-Experimental Structural Optimization Platform) in addition to specific interfaces with the finite element code NASTRAN. A flowchart of the procedure is shown in Fig. 1.

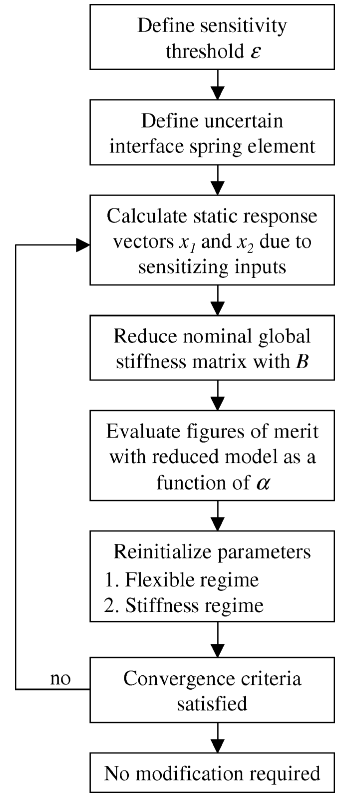

Fig. 1 Flowchart of the reinitializing procedure.

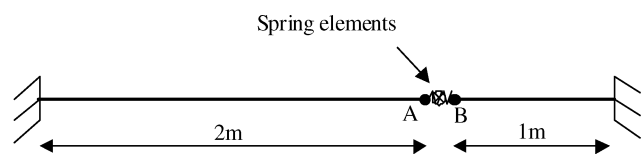

Fig. 2 Definition of the clamped–clamped beam.

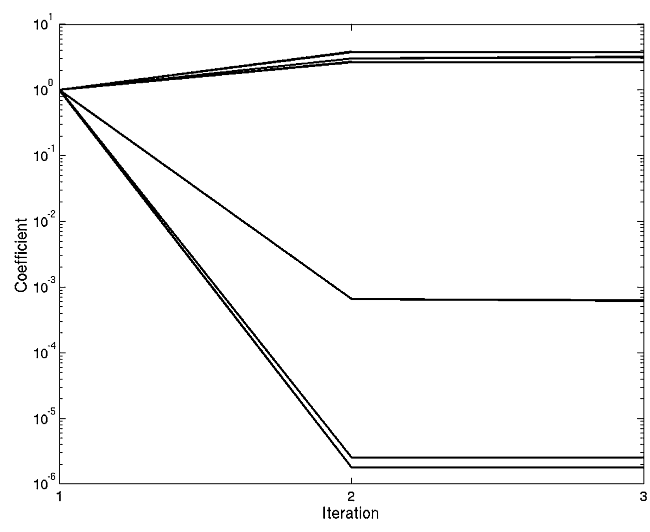

Fig. 3 Evolution of correction coefficients for the clamped–clamped beam.

IV. Numerical Application

A. Clamped–Clamped Beam

To illustrate the proposed methodology, we will consider two examples. The first one is a simple clamped-clamped beam modeled with 60 beam elements and with six connecting elements at two-thirds of its length, one for each nodal DOF. Values of the six springs were initialized to default values without distinction of their role in the model. The clamped–clamped beam is defined in Fig. 2. The two points A and B are at the same location in the finite element model. The characteristics of the material and of the cross section of the beam are as follows: Young's modulus, 7500 MPa; Poisson coefficient, 0.25; density, 9800 kg/m^3; section area, 0.012 m^2; bending moment of inertia, 1.44×10^{-5}, 1.10^{-5} m^4; and torsional stiffness, 2.94×10^{-5} m^4. The results of the reinitialization are summarized in Figs. 3–7. Qualitatively we can make the following remarks:

1) Figure 3 represents the evolution of the correction coefficients with regard to the iteration. Note that two iterations are necessary

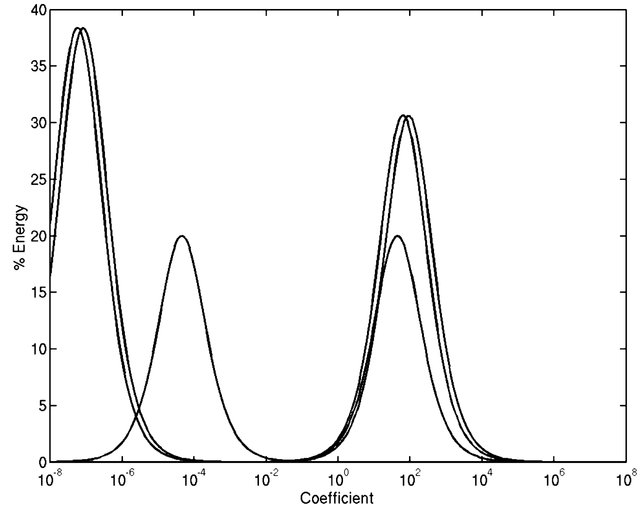

Fig. 4 Initial strain energy ratio for the clamped–clamped beam.

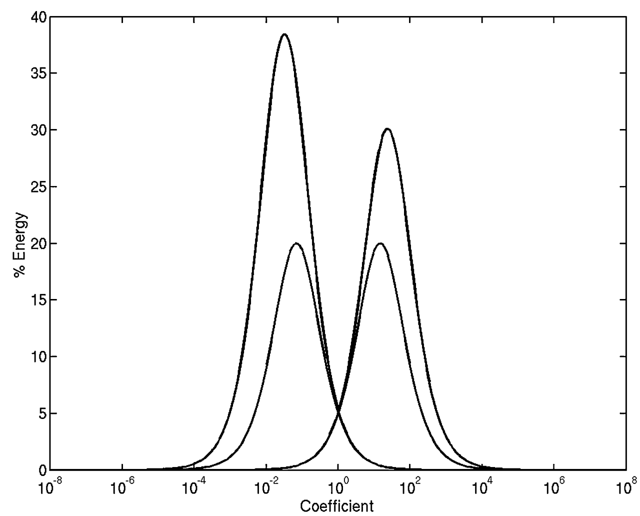

Fig. 5 Final strain energy ratio for the clamped–clamped beam.

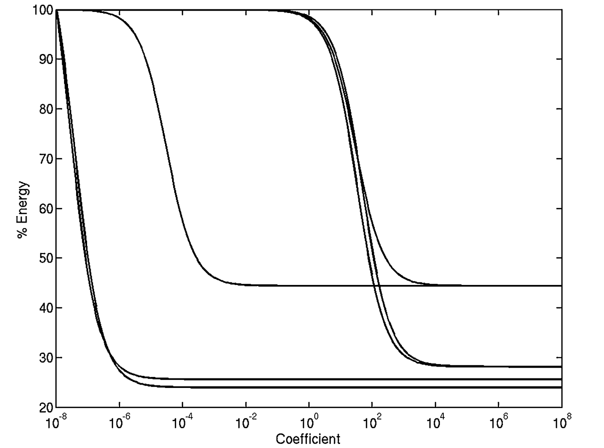

Fig. 6 Evolution of $\mathcal{M}_i^2$ for the clamped–clamped beam (nominal model).

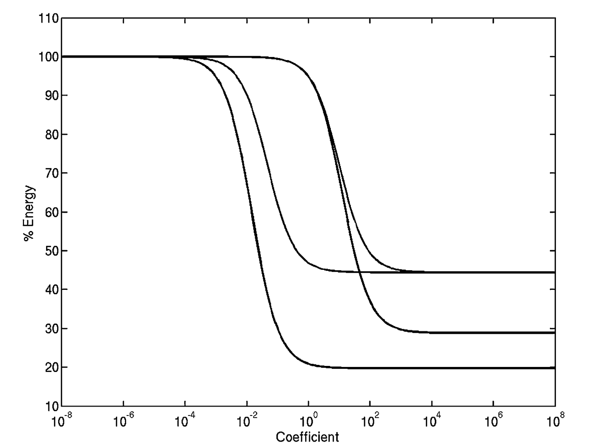

Fig. 7 Evolution of $\mathcal{M}_i^2$ for the clamped–clamped beam (reinitialized model).

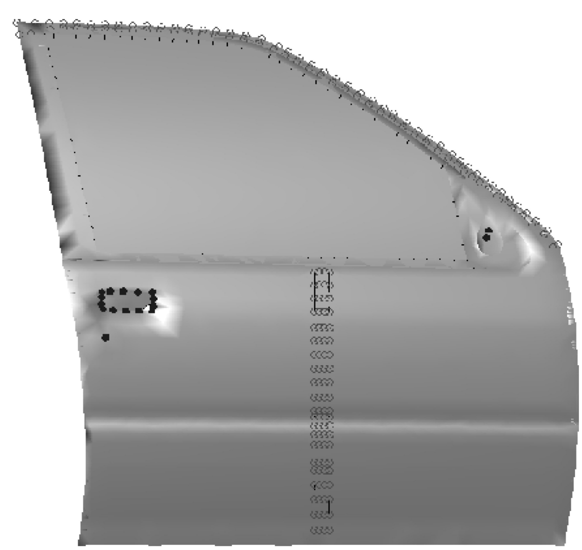

Fig. 8 Finite element model of a car door.

to obtain reinitialized values calculated for an energy sensitivity tolerance of $\varepsilon = 10\%$. The first iteration corresponds to the nominal values. The final correction coefficients correspond to the converged values at iteration 3.

2) Figures 4 and 5 shows the figures of merit $\mathcal{M}_i^1$ as a function of α with a log scale in the abscissa. Each curve corresponds to a different spring element. These figures of merit are measures of the relative energy in a given spring with respect to the total strain energy of the corresponding displacement field. All of these curves are symmetrical and bell shaped, indicating that the relative energy in a given spring, for very low and very high values of the stiffness coefficient, is nearly zero and that a maximum is reached at some intermediate value. The position and value of the peak relative energies depend on the nominal value and topological location of each spring. The final curves have been shifted as a result of the reinitialization of the corresponding stiffnesses. Note that two pairs of curves are superimposed in Fig. 5.

3) Figures 6 and 7 show the evolution of the figure of merit $\mathcal{M}_i^2$ as a function of α. This figure of merit is a measure of the relative energy in a given spring with respect to the maximum total strain energy of the set of displacement fields resulting from $\alpha \in \mathcal{A}$. The monotonically decreasing values of $\mathcal{M}_i^2$ simply reflect the fact that the total strain energy decreases with increasing spring stiffness. In Fig. 7, the translation on the right of the decreasing of the three curves indicates that the final energies in the torsional springs are larger than at the beginning. This is verified by the correction coefficient, which is greater than one for these three springs.

B. Industrial Test Case

The second example is a finite element model of technological interest, in particular a car door, which is shown in Fig. 8 and is composed of nearly 5400 nodes and 6800 finite elements, among which over 1482 are local spring elements (CELAS2 elements in the MSC/NASTRAN library). 779 springs are used to simulate a well-identified rubber joint and will not be modified. The 703 other stiffnesses coefficients of spring elements have been reinitialized without distinction of their role in the model.

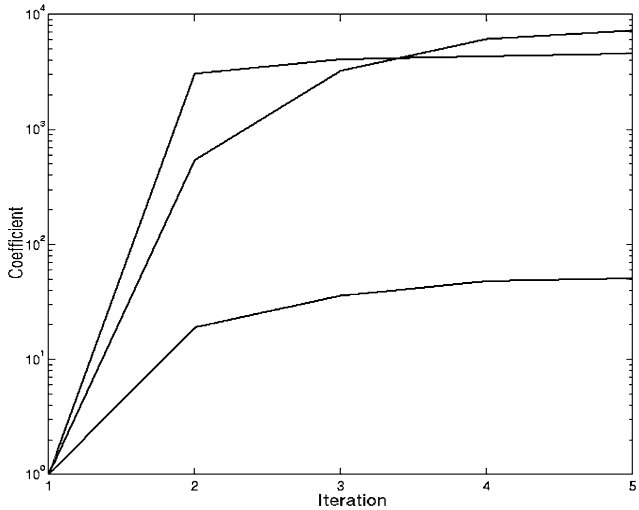

Fig. 9 Evolution of three typical correction coefficients for car door ($\varepsilon = 10\%$).

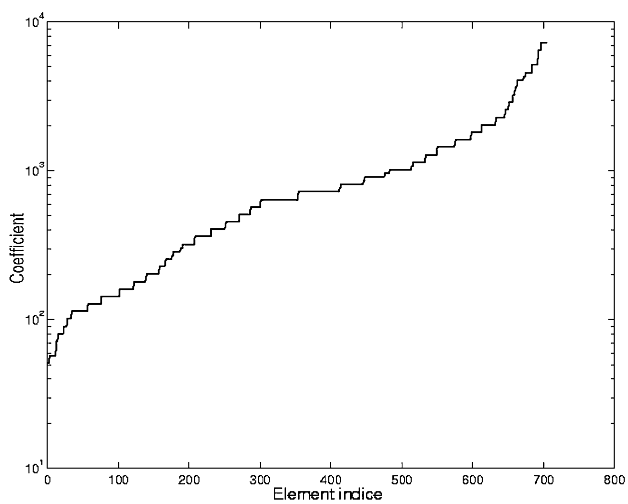

Fig. 10 Correction coefficients for car door ($\varepsilon = 10\%$).

1) Three typical evolutions of the coefficients are shown in Fig. 9 for an energy sensitivity threshold of $\varepsilon = 10\%$. Five iterations are necessary to obtain sensitized values of stiffnesses.

2) Figure 10 represents the final correction coefficients of each selected spring in increasing order.

3) Figure 11 show three typical figures of merit $\mathcal{M}_i^1$. Some of the curves are not bell shaped and concentrate 100% of the strain energy for low coefficients before progressively tending to zero. This behavior is characteristic of springs that are the unique connection between two otherwise independent substructures. Other curves are not bell shaped and concentrate 0% of the strain energy for low coefficients before progressively tending to 100%. This behavior is characteristic of springs for which one of the two nodes is clamped.

4) Figure 12 shows the modal-assurance-criterion (MAC) matrix between the initial eigenmodes (ordinate axis) and the final eigenmodes resulting from the correction coefficients applied to the spring stiffnesses with a threshold of $\varepsilon = 10\%$. To avoid perturbing the nominal modes too strongly, the sensitivity threshold ε must be set to an acceptable value.

5) Figure 13 shows the MAC matrix between the initial eigenmodes (in ordinate) and the final eigenmodes resulting from the correction coefficients obtained with a tolerance threshold of $\varepsilon = 0.1\%$. The eigenmodes are only slightly modified.

6) Evolution of the correction coefficients is shown in Fig. 14. Only three iterations (including computation with nominal values)

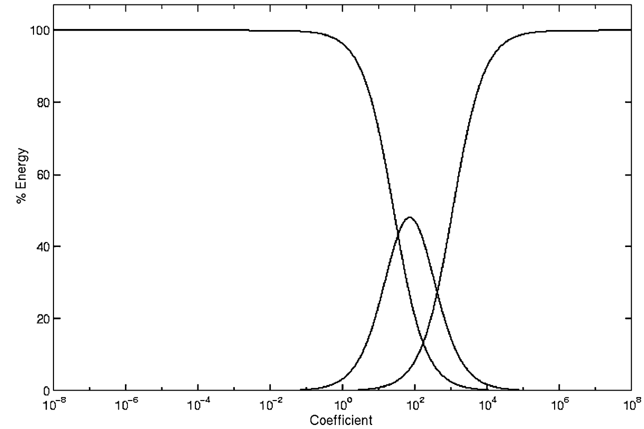

Fig. 11 Typical final strain energy ratios for car door ($\varepsilon = 10\%$).

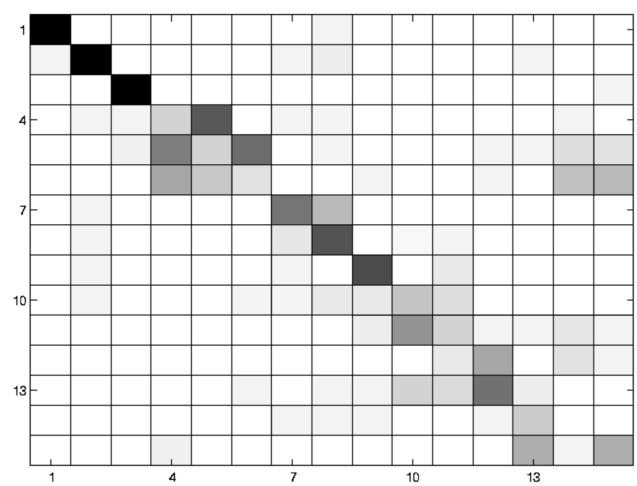

Fig. 12 MAC matrix for car door ($\varepsilon = 10\%$).

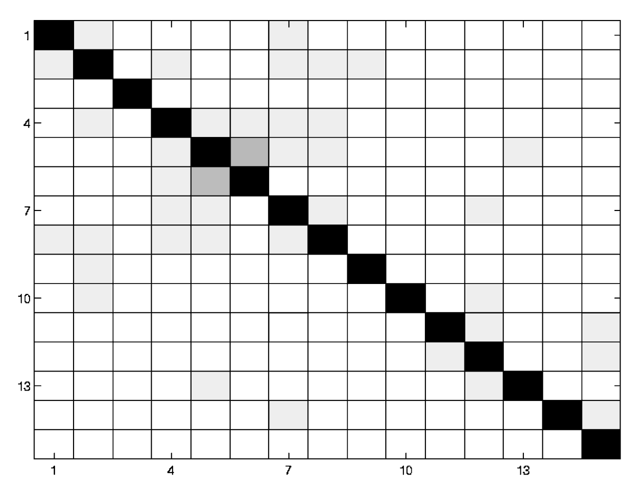

Fig. 13 MAC matrix for car door ($\varepsilon = 0.1\%$).

are necessary to obtain sensitized values and respect convergence criteria for a threshold $\varepsilon = 0.1\%$.

7) The reinitialized stiffness properties cannot be attributed any inherent physical signification other than being better adapted for use in a model optimization or updating procedure. If the stiffness properties are precisely known a priori, they should not be reinitialized.

V. Discussion

Although the basic reinitialization procedure is relatively simple in theory, a number of remarks must be taken into account in order to implement the strategy for use in an industrial context:

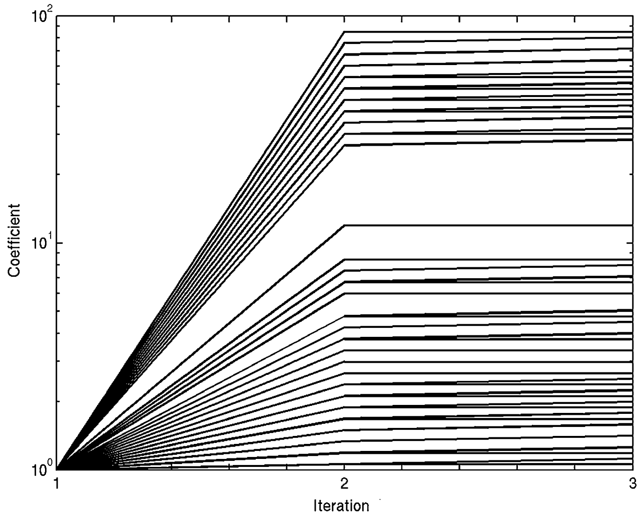

Fig. 14 Correction coefficients for car door ($\varepsilon = 0.1\%$).

1) The model responses are often very sensitive to changes in local stiffness values. The sensitivity threshold ε should be adjusted to avoid a significant perturbation of the nominal eigenmodes.

2) In the case of free structures, the static displacements resulting from the sensitizing inputs can be calculated using the notion of inertia relief forces. Inertia relief allows to simulate unconstrained structures in a static analysis using the inertia (mass) of the structure to resist the applied loadings.[4]

VI. Conclusions

Discretized structural models often make use of local stiffness elements to represent complex joint assemblies. However, the resulting topological simplification renders the determination of the Hookes coefficients difficult, and typically a manual iterative approach is used to estimate initial values in preparation for a parameter optimization or identification procedure. We have proposed a general methodology for automatically reinitializing the Hooke coefficients of local stiffness elements as a function of strain energy distribution in the structure resulting from sensitizing static inputs. This reinitialization is especially important if the spring coefficients are to be used in optimization or updating analyses. The novelty of the strategy lies in the implementation of two ideas. First, a systematic procedure for generating sensitizing static vectors is defined thus providing a reliable measure of joint sensitivity. Second, a simple model reduction procedure is employed thus allowing a significant reduction in calculation cost to be obtained. Two limitations of the proposed procedure. In particular, although the procedure provides new stiffness coefficients that render a given spring sensitive for a specific sensitizing input it does not guarantee that the spring will be sensitized by the displacement fields present within a given frequency range of analysis. This obviously will depend on the deformation shapes. Moreover, the new values do not have any a priori physical meaning, that is to say, it might well be that a given stiffness parameter is truly much stiffer or much more flexible than its surrounding environment.

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